“FANCIFUL” OR GENUINE? BASES AND HIGH NUMERALS IN POLYNESIAN NUMBER SYSTEMS

ANDREA BENDER AND SIEGHARD BELLER
University of Freiburg

Modern Western cultures are based to a considerable extent on writing and numbers. Indeed, numbers are taken as self-evident, even when they greatly surpass the imagination (the stated deficits of many Western state budgets are persuasive testimony of this). Since very high numbers are rarely needed in everyday discourse, they appear predominantly in written form. It may therefore come as a surprise to find that several languages of Polynesian cultures—which, although possessing a rich oral literature, apparently had neither a writing tradition nor a notation for numbers—contain numerals up to which nobody can count, such as 100,000 or, in rare cases, even beyond. For what purposes were such high numbers needed or used? Were they genuine number words or, as two scholars of Hawaiian (Elbert and Pukui 1979:160f.) put it, merely rather “fancifully translated” lexemes that were actually used to poetically indicate great numbers?

A second peculiarity of some Polynesian languages is an evident preference for irregular or mixed bases. According to contemporary dictionaries, decimal systems prevail throughout Polynesia, but evidence of other systems in use before Western influence can be found as well. Examples are apparently irregular ways of counting certain objects in Tongan that emphasise pairs and scores (Bender and Beller, n.d.a); a specific lexeme for 20, *tekau*, in several languages (Best 1906:158, Large 1902, Lynch *et al*. 2002, Smith 1902:216, Tregear 1969:503f.); an allegedly vigesimal system in traditional Māori (Best 1906); and a mixed base 4 and 10 system in Hawaiian (Hughes 1982). All these cases seem to indicate that the traditional Polynesian base was not decimal. This conjecture was advanced, for instance, by Best (1906) and Hughes (1982)—but is it conclusive? Or is it also possible that decimal and mixed base systems were used simultaneously? In other words, to what extent are these number systems indigenous and which parts were introduced in colonial or post-colonial times? And should we assume that mixed bases are a cognitive handicap or might they have served reasonable, practical purposes?

In this article, we present a comparative description and cognitive analysis of Polynesian number systems. Focusing on their bases and extent, we analyse their peculiarities and common patterns. We also pursue a diachronic approach in order to identify which characteristics were inherited from a common linguistic stock, which were developed within the respective cultures and which were introduced by Europeans. Since written documents from pre-colonial times do not exist, we have only two ways of addressing this latter question: on the one hand, by comparing related languages in search of commonalities and differences and, on the other hand, by taking into account terms from Proto-Austronesian, reconstructed for a period of time as long as six millennia ago.

Before we begin our analysis, a few general aspects of numeration systems need to be introduced in order to structure the description of certain aspects and to illuminate their cognitive implications (for a more detailed account see Wiese 2003).

In principle, a one-dimensional system would be sufficient for the representation of natural numbers, that is a system with a distinct lexeme for each number. However, since this is not efficient for large numbers, many languages apply a two-dimensional system of base and power (see Zhang and Norman 1995). In the English system, for instance, the base is 10. Larger number words, for instance 3482, are generated according to the multiplication and addition principle, that is by adding the multiples of the base raised to various powers: in this example as 'three thousand' ($3x10^3$), 'four hundred' ($4x10^2$), 'eighty' ($8x10^1$) and 'two' ($2x10^0$). A strict decimal system necessitates nine words for the basic numbers 1 to 9, and one word each for the base and its higher powers (10, 100, 1000 …). A word for zero, essential in strict place-value notations, is not required as part of the number system in natural languages; non existing powers, such as the tens in 402, can be simply left out when saying "four hundred and two".

The way in which a number system is structured affects the way in which people operate with it cognitively (e.g., Dehaene 1997, Wiese 2003). Irregularities in the composition of number words, for instance, slow down their acquisition and impede particular counting or calculating strategies (Geary *et al*. 1996, Miller *et al*. 1995). Base size, on the other hand, comes with a cognitive trade-off: the larger a base is, the more efficient it is for encoding and memorising big numbers, yet the smaller a base, the more it facilitates calculation owing to smaller addition and multiplication tables (Zhang and Norman 1995). A medium-sized decimal system, for instance, requires the memorisation of 55 products (for multiplications up to the base); the larger vigesimal system requires 210, the smaller quinary system only 15.

As languages encompass only a finite set of lexemes, the system of regularly composed number words is also limited in most natural languages. The limiting number (L) is defined as the next number beyond the highest possible composition (Greenberg 1978:253), usually one power higher than the largest numeral. In a decimal system with "hundred" as the highest numeral, for instance, the limiting number is 999 + 1, which is a thousand.

While this limiting number can serve as an indicator for the extent of the respective number system, it does not depend on mathematical comprehension (see Ifrah 1985), but only on the concern with numbers in the respective culture (see Ascher 1998:5). In principle, it is possible to extend the limit of a number system beyond L, either by saying "… and one more" or by multiplying powers of the base as in English "ten thousand" and "hundred thousand". A third option will be discussed further below.

In the following descriptions of Polynesian number systems, we use the term "number" for numerical values, "numeral" for basic number words, "base" for that number that recurs in powers and "mixed base" to refer to those systems that appear to deviate from a strictly decimal base. Our descriptions and analyses are organised chronologically. We start by outlining what we know about the number system of the Polynesians' Austronesian ancestors and of remote, yet related, languages, and then turn to those elements shared by most contemporary Polynesian languages. Based on an overview of their commonalities, we look in greater detail at a few selected cases and their respective numeration principles. In drawing our conclusions concerning the base and extent of these number systems, we try to prove that—even without notation—Polynesian cultures did indeed have use for high numbers. By speculating on how they might have handled these, we argue that the questions of base and extent are inextricably linked.

THE AUSTRONESIAN HERITAGE

About 6000 years ago, a group of seafaring people with one common language, originating from Southern China, set off for new shores in outrigger canoes. Over the next millennia, they spread out over a vast area from Madagascar in the West to Rapanui (Easter Island) in the East, diversifying both culturally and linguistically.[1] However, despite this diversity, their present-day descendants share distinct cultural traits and linguistic characteristics that are used to define them as Austronesians (Bellwood *et al*. 1995). Within the Austronesian language family, the contemporary Polynesian languages are geographically the most eastern, comprising the Oceanic Subgroup (see Fig. 1). In order to determine the extent to which present-day Polynesian number systems are specifically Polynesian, we search for common ancestry by comparing selected Austronesian languages of this vast region and reconstructing ancestral lexemes.

With approximately 1200 languages, the Austronesian language family is the largest in the world (Tryon 1995:6). Eighty of these, taken from all subgroups and major branches, are gathered in the *Comparative Austronesian Dictionary* (Tryon *et al*. 1995), which provides one of our main data bases. We

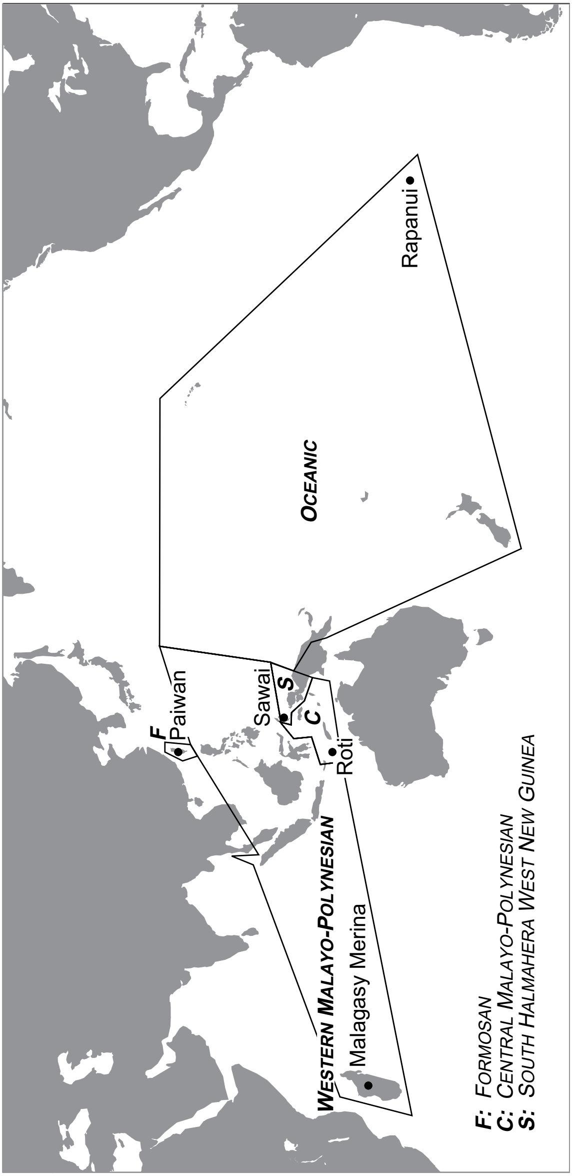

Figure 1. The Austronesian language family and its major groups (according to Kirch and Green 2001:40, Tryon 1995:7). All languages except for the Formosan constitute the Malayo-Polynesian group. Subgroups are denoted in capital letters; the languages chosen for comparison (cf. Table 1) are marked by a dot.

have selected five of these in order to demonstrate how close and widespread the basic numerals still are among the Austronesian languages and to assess the reliability of the reconstruction of the Proto-Austronesian (PAN) terms (see Table 1). We chose the following examples of contemporary languages in order to cover the five geographical areas identified by Kirch and Green (2001:40) and Tryon (1995:7) as the major Austronesian subgroups: Paiwan (Formosan) on Taiwan, Malagasy Merina (Western Malayo-Polynesian) on Madagascar, Roti (Central Malayo-Polynesian) and Sawai (South Halmahera West New Guinea) in Indonesia, and Rapanui (Oceanic) on Easter Island in the Pacific (see Figure 1).

If we look at the Proto-Austronesian forms, we find a set of numerals from 1 to 10, and in Proto-Oceanic a numeral for 100, **Ratu*(*s*), which strongly indicates a decimal system with a limiting number of at least 1000. This pattern still prevails in most parts of contemporary Austronesia, although with some variation in the Oceanic subgroup. While most of the Oceanic languages, particularly in Polynesia and Micronesia, use decimal number systems, the Melanesian languages reveal greater heterogeneity. There, we find a mix of decimal and quinary (base 5) systems, the latter being most widespread in Vanuatu, New Caledonia, and in some areas to the west (Lynch *et al*. 2002:39,72).

With regard to the limiting number, the prevailing decimal and the exceptional quinary systems also diverge. Although even Proto-Malayo-Polynesian most likely had a term for hundred (**Ratús*), the numbers above ten were clearly not in wide use in all settlement areas (Lynch *et al*. 2002:72), and numerals for higher numbers were given up together with the decimal system in some languages. While the highest numeral in (semi-)quinary systems is usually 20 (i.e., "one man" with all his fingers and toes), the decimal systems found in the remaining parts of Oceania contain numerals for 100 and 1000 (Tryon *et al*.1995, IV:50-53), and reach as far as 1,000,000,000 in some Micronesian languages (Harrison and Jackson 1984) and 2,000,000,000 in Mangareva (Lemaître 1985).

The similarity in number systems already apparent in most Oceanic languages becomes even more striking if we look at the Polynesian languages.

THE GENERAL POLYNESIAN NUMBER SYSTEMS

The distinctive traits of Polynesian cultures began to take shape 2600 years ago after a group of Oceanic-Austronesian-speakers travelling east arrived and settled in the core area of Western Polynesia around 3000 years ago. From here, some moved back west to what are now called the Polynesian Outliers, and others continued east to Central Polynesia and, a few centuries later, to

Table 1: Numerals in Austronesian languages, both reconstructed and contemporary.

Number	PAN	Paiwan	MGY	Roti	Sawai	PO	Rapanui
1	*a-sa, *’e+sá, *i-sá, *sa-, *ta+sa	ita	irai	esa	pu-sɔ	*ta-sa, *sa-kai, *tai, *kai	tahi
2	*d$_3$uSá	ḍusa	rua	dua	pɛ-lu	*rua	rua
3	*tĕlú	cəḷu	telu	telu	pɛ-tel	*tolu	toru
4	*Sĕ(m)pát	səpac	efaṭ^{r}a	ha	pɛ-fɔt	*pati, *pat	hā
5	*limá	ḷima	dimi	lima	pɛ-lim	*lima	rima
6	*’ĕném	unəm	enina	ne	pɛ-wonɛm	*onom	ono
7	*pitú	picu	fitu	hitu	pɛ-fit	*pitu	hitu
8	*walú	aḷu	valu	falu	pɛ-wal	*walu	va’u
9	?*siáw	siva	sivi	sio	pɛ-popɛt	*siwa	iva
10	*púluq	ta-puḷuḳ	fulu	sana-hulu	yɔfɛ-sɔ	*sa[-nga]-puluq	‘angahuru
100		taiday	zatu	natu-n esa	witɛn-čɔ	*Ratu(s)	rau
1000		kuzul^y	*arivu*	lifu-n esa	čalɛn-čɔ		piere

Abbreviations: PAN=Proto-Austronesian, MGY=Malagasy Merina, PO=Proto-Oceanic.
Sources: PAN, Paiwan, MGY, Roti and Sawai are taken from Tryon (1995:18) and Tryon *et al.* (IV:27-53), PO from Lynch *et al.* (2002:72) and Rapanui from Englert (1978). *Notes:* Loan words (in this case from Malay) are given in italics; reconstructed terms are underlaid in gray.

Easter Island, Hawai‘i and New Zealand (Kirch and Green 2001:79-81; see Figures 1 and 2). During these millennia, two interacting centres may be distinguished: the first in the Western Polynesia core area, where Polynesian culture and language developed and from where the Outliers were populated, and later the Central Eastern Polynesian core area from where the edges of the Polynesian Triangle were settled.

Despite their differentiation during the last two millennia, Polynesian languages and cultures have preserved considerable similarities right up to the present, with the greatest linguistic differences setting apart Tongan (and Niuean) from the rest (see Figure 2).

This overall similarity, particularly in the words for numerals, caught the attention of Western observers from the very beginning of culture contact. As early as 1839, John Davis stated in the *Hawaiian Spectator* (cited in Hughes 1982:253) that the numerals for 1 to 10 are very close to each other in Tahitian, Marquesan, Rapa, Rarotongan, New Zealand Māori (henceforth Māori), Rapanui and Hawaiian (see also Tregear 1969). Comparing the numerals from the nine contemporary Polynesian languages (Table 2), including five of those considered by Davis along with four non-Eastern Polynesian languages (Tongan, Samoan, Rennellese and Nukuoro), we can still confirm his impression.

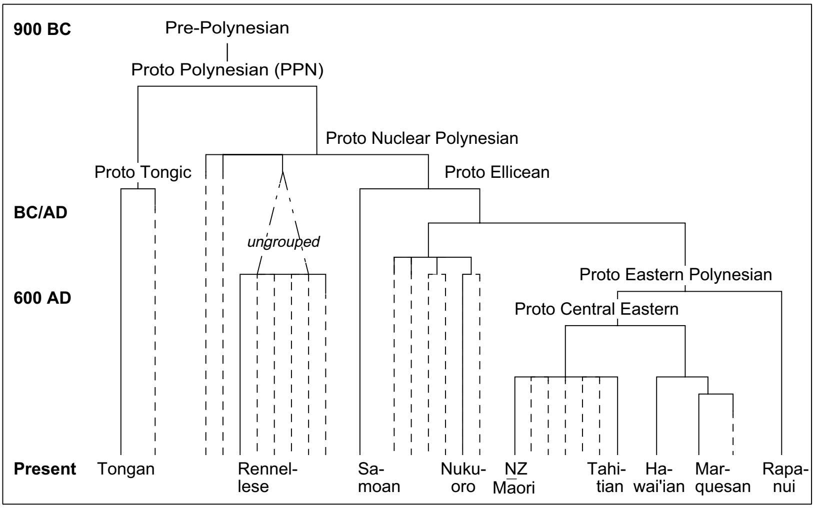

Figure 2. A family tree of Polynesian languages (adapted from Kirch and Green 2001:61). Only those languages referred to in the text are explicitly mentioned, the remaining ones are indicated by interrupted lines.

Table 2: Numerals in contemporary Polynesian languages (modern systems).

Number	Tongan	Samoan	Rennellese	Nukuoro	Marquesan	Tahitian	Rapanui	Hawaiian	Māori
(0)	noa	*selo*				‘aore	‘ina	‘ole	
1	taha	tasi	tasi / tahi	dahi	tahi	hō‘ē, tahi	tahi	kahi	tahi
2	ua	lua	gu(a)	lua	‘ua	piti	rua	lua	rua
3	tolu	tolu	togu	dolu	to‘u	toru	toru	kolu	toru
4	fā	fā	hā	haa	hā / fā	maha	hā	hā	whā
5	nima	lima	gima	lima	‘ima	pae	rima	lima	rima
6	ono	ono	ono	ono	ono	ono	ono	ono	ono
7	fitu	fitu	hitu	hidu	hitu / fitu	hitu	hitu	hiku	whitu
8	valu	valu	bagu	valu	va‘u	va‘u	va‘u	walu	waru
9	hiva	iva	iba	siva	iva	iva	iva	iwa	iwa
10	hongofulu	sefulu	angahugu	hulu	‘onohu‘u/‘umi	‘ahuru	‘angahuru	‘umi	tekau

continued over

Number	Tongan	Samoan	Rennellese	Nukuoro	Marquesan	Tahitian	Rapanui	Hawaiian	Māori
10^2	teau	selau	gau	lau	*hānere*	*hānere*	*hānere*	*hanele*, *haneri*	rau
10^3	afe	afe	noa	mano	*tautini*	*tauatini*, *tautini*	*ta'utini*	*kaukani*, *tausani*	mano
10^4	mano	[sefulu afe]	bane	semada					[tekau mano]
10^5	kilu	mano	tuia	seguli					[rau mano]
10^6	*miliona*	*miliona*	nimo	seloo				*miliona*	

Sources: Tongan numerals are from Churchward (1953), Samoan from Milner (1966), Rennellese from Elbert (1988), Nukuoro from Carroll and Soulik (1973), Marquesan from Lynch *et al.* (2002:869) and Lemaître (1985), Tahitian from Lemaître (1973, 1985), Rapanui from Tryon *et al.* (1995, IV), Hawaiian from Pukui and Elbert (1986) and Māori from Biggs (1990a) and Bauer *et al.* (1997:275). For Proto-Polynesian numerals see Clark (1999).
Notes: Prefixes are omitted for easier comparison. Loan words (in this case from English) are given in italics. Although the Polynesian language family comprises close to 30 languages (Tryon 1995, I:15), only nine are included in this overview; we may consider the selection to be balanced, however, as for most groups at least one member could be considered (see Figure 2).

For each of the numbers from 1 to 9, lexical coincidence (within the range of regular sound shift) appears in not less than eight of the nine languages, and often in all of them. If we look at the diverging terms in greater detail, we still find traces of the common Polynesian ancestry even there. In the case of Tahitian, which contains three diverging terms (i.e., *piti* for 2, *maha* for 4 and *pae* for 5), additional "old" terms are reported that confirm the linguistic relationship at first glance: *rua*, *fā* or *hā* and *rima* respectively (Lemaître 1973, 1985; Tryon *et al.*1995, IV:33-36). The term for 10 has undergone the biggest changes but is still recognisable in Tongan, Samoan, Rennellese, Nukuoro, Marquesan, Tahitian and Rapanui. In Māori, the indigenous term for 10, *ngahuru*, which paralleled other Polynesian numerals, was replaced by Europeans with *tekau* (Best 1906:151). The Hawaiian term for 10, *'umi*, also differs from the shared Polynesian stock, but a lexeme for 10 close to the common term can still be found in Hawaiian to denote the traditional '10-day week', *anahulu* (Hughes 1982:254).

The variability of the lexeme for zero is not surprising. Zero is not required in oral number systems (Greenberg 1978:255) and was most probably introduced with a written place-value notation that required a digit for 0. It is interesting to note that in most cases an indigenous term roughly denoting 'nothingness' was chosen to express the concept of zero, whereas for other numbers (such as 100 or 1000 in Marquesan, Tahitian, Rapanui and Hawaiian) English loan words are in use.

When comparing the numerals beyond ten, the pattern initially appears to be somewhat more fragmented. Four languages seem to have a limiting number of 100 (with English loan words used from this point on), while others yield indigenous lexemes up to 100,000 or even beyond. This is surprising when we consider that the first group includes languages of highly stratified societies, such as Hawai'i and Tahiti, that probably had a greater need for high numbers, at least for systems of resource redistribution. It would also be astonishing if the numeral for 1000 (*mano*) were present both in Western Polynesian languages and in Māori, but not in the languages of the second language and culture centre of Tahiti and the Marquesas, from whence Māori derived.

However, if we include the indigenous or so-called "archaic" terms reported for Marquesan, Tahitian, Rapanui and Hawaiian, these oddities give way to a more coherent picture (Table 3). One numeral appearing in every language is *teau / selau / gau / lau / 'au / rau*, whose differences are attributable to sound shifts (see also Clark 1999). This numeral always denotes the second power of the base, i.e., 100 in Tongan, Samoan, Nukuoro, Rennellese, Tahitian, Rapanui and Māori. In Sout Eastern Marquesan it stands for 200 and in North Western Marquesan and Hawaiian for 400. A second

Table 3: Traditional or “archaic” numerals for the powers of the base in Polynesian languages.

Power of base	Tongan	Samoan	Rennellese	Nukuoro	Marquesan	Tahitian	Rapanui	Hawaiian	Māori
10^1	hongofulu	sefulu	angahugu	hulu	‘onohu‘u / ‘umi	‘ahuru	‘angahuru	‘umi	ngahuru
10^2	teau	selau	gau	lau	‘au	rau	rau	lau	rau
10^3	afe	afe	noa / **mano** / ahe / kiu	**mano**	**mano**	**mano**	piere	**mano**	**mano**
10^4	**mano**	?	bane / kiu	semada	tini	manotini	**mano**	kini	[tini]
10^5	kilu	**mano**	tuia	seguli		rehu		lehu	
10^6			nimo	seloo		‘iu		nalowale	
10^7				sengaa					
10^8				semuna					

Sources: Tongan numerals are from Churchward (1953), Samoan from Milner (1966), Rennellese from Elbert (1988), Nukuoro from Carroll and Soulik (1973), Marquesan from Dordillon (1904) and Lemaître (1985), Tahitian from Tregear (1969) and Lemaître (1985), Rapanui from Englert (1978), Hawaiian from Hughes (1982) and Māori from Best (1906). *Notes:* Prefixes are omitted for easier comparison. The highest numeral common to all languages, mano, is highlighted; numerals diverging in value from a strictly decimal pattern are underlaid in gray.

numeral common to most languages, albeit with a diverging value attached to it, is *mano*. It generally denotes a higher power of ten, ranging from 1000 in Nukuoro, Tahitian and Māori through 10,000 in Tongan and Rapanui and up to 100,000 in Samoan. In addition, it reappears in traditional Marquesan and Hawaiian, where it refers to the numbers 2000 and 4000 respectively (Dordillon 1931, Hughes 1982). In Rennellese, it is not part of the general system but appears in two specific systems, referring to 1000 coconuts or 100 piles of bananas (Elbert 1988). And even outside Polynesia, namely in Lewo on Vanuatu, we can find *manu* denoting 1000 (Tryon *et* al.1995, IV:52). Other numerals are spread with less frequency, such as *afe/ahe* or derivates of *tini* (*manotini* and *kini*) and *rehu*/*lehu*. This comparison reveals that, despite the partial change in numerical value, a majority of the Polynesian languages shared terms for the higher powers of their base, and, at least in some cases, the limiting number of the system was large.

However, variation in extent is still remarkable. In Māori, for instance, the limiting number is 10,000. Other languages, such as Tahitian or Rennellese, had numerals for up to 1,000,000, and in Nukuoro, a language spoken by fewer than 1000 people on a Polynesian outlier in the Caroline Islands, we even find terms for up to hundreds of millions (*semuna*). This exceptionally high range might be owing to the influence of surrounding Micronesian languages, which are renowned for their high limiting numbers (Harrison and Jackson 1984, Lynch *et al*. 2002:39). Tongan, Samoan, Tahitian and Hawaiian, however, present clear examples of the Polynesian concern with high numbers.

The Hawaiian case also reveals a second peculiarity. In its indigenous number system, the numerals did not apply to the pure powers of ten, and so were replaced with English loan words from 100 onwards, but to the powers of ten times four, such as 400, 4000 and so on. A similar mixed base system also appears to have prevailed in Northern Marquesan (Dordillon 1904, 1931).

With regard to the base, we therefore have evidence for two divergent hypotheses. On the one hand, given that most of the selected languages have similar numerals for the numbers 1 to 10 and comparable terms for at least one further power of ten, the hypothesis might be that they also shared a decimal base. On the other hand, we have evidence for apparently mixed base systems in at least some of these languages before European influence. Some scholars have therefore argued that decimal systems were introduced (or rather re-introduced) by missionaries to replace these mixed base systems (Bauer *et al*. 1997:289, Best 1906, Hughes 1982, Large 1902). In order to examine both the actual extent of the number systems as well as the hypotheses concerning their bases, we need to look more thoroughly at some of these number systems.

SPECIFIC NUMBER SYSTEMS

For the sake of simplicity we have only depicted the regular aspects of Polynesian number systems in Table 2, as they are reflected in modern-day usage. However, this simplification betrays the very interesting and noteworthy peculiarities of some of these systems, which we outline in detail for Māori, Hawaiian and Rennellese in the following sections. We begin with Māori, for which the parallel use of different systems is documented and a (semi-)vigesimal base has been claimed. An apparently mixed base 10 and 4 system as well as indicators for a decimal base can be found in the Hawaiian system, which is also characterised by one of the highest limiting numbers. Finally, the Rennellese systems are among the most elaborate in Polynesia, encompassing 14 different counting modes for specific objects, some of which seem to apply mixed bases as well. At the end of this section, we address the question of whether these systems are unique or reflect a common Polynesian pattern.

The Māori Number Systems

Turning to New Zealand, an "outpost" of Polynesian cultures, we are confronted with a perfectly coherent, decimal number system in modern Māori. However, the picture turns out to be more complex if we take into consideration old references to the indigenous system before European influence.

Since the publication of Best's (1906) work, a broadly shared general conviction is that the traditional Māori number systems were based on twenty:

> … Māori formerly had two parallel counting systems, counting by ones (normal for people), and counting in pairs (normal for game, etc), both involving a base twenty system. The base twenty system was replaced by the modern decimal system after European contact. (Bauer *et al*. 1997:289)

The two main pieces of evidence for this conjecture are a special role of 20 with a distinct numeral for it (*tekau*) and a prefix, *hoko*-, allegedly multiplying the subjoined numeral by 20 (Bauer *et al*. 1997:284,288; Williams 1988:57). However, if we scrutinise the source cited for these remarks and the data presented there (Best 1906), this statement turns out to be questionable.

One problem, which clearly applies to all of our investigations to an even greater extent and which Best himself admits, is that at the time of his research the proposed shift had already taken place: "The older generation of living Natives can only recall the old-time numerical terms by an effort of memory; indeed, some have forgotten many of them. The younger generation know practically nothing of these matters” (Best 1906:160). This led to a range of “confusions” (Best 1906:160,171) and renders both Best’s and our own interpretations a little speculative. But this should not be an issue here. Best was the first and strongest advocate of a vigesimal system, and it should be satisfactory if we can show that his data, supposed to support his view, can instead serve to strengthen our rejection of it.

Old Māori contained two modes of counting: *tatau takitahi* ‘counting singly’ and *tatau tōpū* ‘counting pairs’. These two modes largely—but not exclusively, as the terms between 20 and 200 show—mapped onto two main systems, which Best terms “single” and “binary or dual”. The dual system was used for certain kinds of objects, but the single system is also said to have been commonly used (Best 1906:154). A third system, restricted to counting people (*tatau tangata*), is similar to the single system in the lower ranges (except for different prefixes required by the numerals between 2 and 19 inclusive) and similar to the dual system in the upper ranges (see Table 4).

When counting an odd number in the pair mode—as, for instance, when collecting birds from traps—single objects were referred to only in the result, not in the process (Best 1906:164), by listing the odd one with *tautahi* as in:

hoko-	toru	e waru	pū	tautahi	
10-fold	3	+ 8	brace	single/odd one	
{[(10 x	3)	+ 8]	x 2 }	+ 1	= 77.

Sometimes, the fowler even tried to avoid obtaining odd numbers by simply waiting for more prey (Best 1906:167).

With *mano* (1000) as the highest numeral, the limiting number is ten thousand in the single mode and twenty thousand in the pair mode. However, according to Best (1906:167f.), Māori conceived of *mano* as the limiting number up to which they would readily count, while for the amounts beyond this they would speak rather of *tini* (great number, multitude).

Best (1906:158) emphasises that traditional Māori numeration was not decimal and that decimal patterns were unfamiliar to the Māori before European influence. Instead, he identifies dual and vigesimal patterns. What he regards as dual is the custom of counting certain objects in pairs, while vigesimal or “semi-vigesimal” are those numerals that are composed with *hoko-* (Best 1906:171). When multiplying the subjoined numeral by 10 in the *takitahi* (single) mode, he considers the *hoko*-terms as semi-vigesimal, and when multiplying by 20 in the *tōpū* (pair) mode, he considers them vigesimal (Best 1906:171). We cannot agree with his interpretation for at least three reasons.

First, even if his conclusions were correct, "vigesimal" would be an inappropriate term for describing the Māori number systems from a mathematical point of view. A vigesimal system requires a pattern that emphasises not just twenty itself or multiples of twenty, but a recurrence of twenty in powers, that is at $20^1 = 20$, $20^2 = 400$, $20^3 = 8000$ and so on. What we find instead is a cyclic pattern at $2\text{x}10^1 = 20$ (*tekau*), $2\text{x}10^2 = 200$ (*rau* [*tōpū*]) and $2\text{x}10^3 = 2000$ (*mano* [*tōpū*]). If at all, this might be rather termed a mixed base 2 and 10 system.

Second, Best (1906:159) argues that in the single and the person system, the odd tens between 20 and 200 are composed as a multiple of twenty with an added ten. However, if we look at these composites in greater detail, we find that in none of the three traditional systems is the lexeme for 20 (*tekau*) itself used as a factor to compose them. In the dual system, which applies counting in pairs most strictly and thus more than the others emphasises 20 as a supplementary base, the term *tekau* is even missing altogether.

Third, what appeared to be the strongest evidence in Best's argument, that the prefix *hoko*- multiplies a joined numeral by 20, turns out in fact to have been a multiplier by 10 used with pairs of things, as the number terms for 40 to 180 (in all three traditional systems) clearly show.[2] Only in the *tōpū* (pair) mode is it the case that, for instance, *hokotoru* (literally *hoko*-3) equals 60. Correctly, the number would have to be glossed as *hokotoru tōpū* = 'hoko-3 in pairs' = (10x3) x 2 = 60. Even when *tōpū* ('pair') is not added to the number term, as is usually the case in the dual system, the 'pair' is always understood, whereas if the counting proceeded in the *takitahi* (single) mode, it had to be made explicit (Best 1906:161).[3]

We therefore propose that what at first glance appeared to be a vigesimal system in Māori is in principle a decimal system operating with pairs of objects instead of single objects in some cases. Best himself provides support for this view that a decimal base conception was not at all unfamiliar to Māori when he describes abbreviation (1906:169) and the rounding down of composed number words (1906:171f.).

Instead of constituting a base, the emphasis on the number 2 might, then, rather refer to the pair as the main counting unit. This assumption is supported by the observation that in certain instances the value of the counting unit could change as the Māori term *pū* (usually translatable as 'pair', but also referring to 'bunch, bundle, heap or stack'; see Williams 1988) did not always exactly refer to two objects. Especially when counting small birds such as the *kōkō* or *tūī* (parson bird)—items that were generally counted in the pair mode—a *pū* consisted of four or even six animals (Best 1906:166,172). Such a change in numerical value would be inconceivable for a fundamental base.

Table 4: Māori numerals (modern and traditional systems).

	Modern system	Traditional systems		
Number	decimal	*takitahi* (single mode)	*tangata* (person mode)	*tōpū* (pair mode)
1	(ko)tahi	(ko)tahi	(ko)tahi	
2	(e) rua	(e) rua	(toko)rua	(ka) tahi pū
3	(e) toru	(e) toru	(toko)toru	
4	(e) whā	(e) whā	(toko)whā	(ka) rua pū
5	(e) rima	(e) rima	(toko)rima	
6	(e) ono	(e) ono	(toko)ono	(ka) toru pū
7	(e) whitu	(e) whitu	(toko)whitu	
8	(e) waru	(e) waru	(toko)waru	(ka) whā pū
9	(e) iwa	(e) iwa	(toko)iwa	
10	**tekau**	ngahuru	(ti)ngahuru	(ka) rima pū
12	tekau ma rua	ngahuru ma rua	(ti)ngahuru ma rua	(ka) ono pū
20	(e) rua tekau	**tekau**	**tekau** / **hoko**rua takitahi	ngahuru pū
30	(e) toru tekau	**tekau** maha ngahuru	**tekau** maha ngahuru	ngahuru pū e rima pū
40	(e) whā tekau	**hoko**rua tōpū/ **hoko**whā takitahi	**hoko**rua tōpū / **hoko**whā takitahi	**hoko**rua
50	(e) rima tekau	**hoko**rua [tōpū] ngahuru takitahi*	**hoko**rua [tōpū] ngahuru takitahi	**hoko**rua e rima pū
100	(ko)tahi rau	**hoko**rima tōpū/ (ko)tahi rau takitahi	**hoko**rima tōpū/ (ko)tahi rau takitahi	**hoko**rima

continued over

	Modern system	Traditional systems		
Number	decimal	*takitahi* (single mode)	*tangata* (person mode)	*tōpū* (pair mode)
200	(e) rua rau	(e) rua rau (takitahi)	(ko)tahi rau tōpū / (e) rua rau (takitahi)	(ko)tahi rau
300	(e) toru rau	(e) toru rau (takitahi)	(ko)tahi rau ma rima tōpū / (ko)tahi rau **hoko**rima	(ko)tahi rau **hoko**rima
400	(e) whā rau	(e) whā rau (takitahi)	(e) rua rau tōpū	(e) rua rau
1,000	(ko)tahi mano	(ko)tahi mano	(e) rima rau tōpū	(e) rima rau
2,000	(e) rua mano	[tini]	(ko)tahi mano tōpū	(ko)tahi mano
10,000	(ko)tahi tekau mano			
100,000	(ko)tahi rau mano			

Source: Adapted from Best (1906).
Notes: Prefixes have been put in brackets for easier comparison. The term *tekau* and the prefixed *hoko-* are highlighted in the traditional systems as they are the key arguments for Best's proposition of a shift from vigesimal to decimal system. The term *takitahi* refers to 'counting singly', while *tōpū* indicates 'counting in pairs'.
* Actually, 50 is denoted as *hoko**toru** ngahuru takitahi* in Best (1906:174, emphasis added); however that must, according to both his translation and the logic of the system, be a typographical error.

A possible rationale for the mixed bases 2 and 10 is indicated by a proverb reported by Best (1906:156f.) that refers to old people eating tough food: *ngahuru kei runga, ngahuru kei raro* 'still ten above, still ten below'. Accordingly, the absolute number of teeth required is twenty, or ten in each jaw (the upper and the lower). However, only in a pair—that is, with a partner on the complementary side—are they useful. Besides emphasising the number 10, this proverb also nicely reflects the Māori concern with symmetry. The same concern can be identified in the variety of terms for pairs and in the dual system used for certain objects (Best 1906). It can also be identified in the preference for even numbers in architecture and decoration (Ascher 1998:171f., Hanson 1983, 2004), for instance, as reflected in the custom of putting even numbers of rafters on either side of a roof so as to avoid bad luck. This concern with symmetry is so predominant that it can be called the "organizing principle… in much of Māori myth, religion, social life, and economics" (Ascher 1998:171).

Best is never explicit about the category of the objects that are counted in pairs in Māori. He refers to them as "game, etc." (e.g., 1906:150,154) or "game, fish, etc." (e.g., 1906:175)—except for one instance where he includes baskets of food (1906:163) and another where he explicitly excludes baskets of sweet potato (1906:172). However, the category seems to have included only products of subsistence and only those that were important—a pattern that will recur in the languages analysed in the following sections.

The Hawaiian Number System(s)

Like the Māori number systems, the Hawaiian system seems to contain decimal and non-decimal elements. Once again, it is therefore necessary to weigh the question of Western influence against the possibility of parallel application in pre-colonial times.

The number system that is in use in contemporary Hawaiian appears to be a regular base ten system. The first nine numerals reflect the common Polynesian lexemes, as depicted in Table 2. The lexeme for 10, *'umi*, differs, but is not without parallel; it can be found, for instance, in Rennellese *kumi* = '10 fathoms, puddings or bags of taro tubers' (Elbert 1988) or in Tongan *tekumi* = '10 fathoms' (Churchward 1953). The number word for 20, *iwakālua*, is irregularly composed and its etymology—apparently containing the numerals for 9 (*iwa*) and 2 (*lua*)—is unclear (Elbert and Pukui 1979:159). The tens between 20 and 100 are, in the modern system, generated regularly with a specific term for 10 (*kana*) multiplied by a single numeral, as in *kana-kolu* (10 x 3), *kana-hā* (10 x 4), and so on. From 100 onwards, English loan words are used for the powers of ten (see Table 5).

Table 5: Hawaiian numerals (modern base 10 and traditional base 4x10 system).

Number	B10 expression		B4x10 expression		Number	B10 expression		B4x10 expression	
1		kahi							
2		lua			20	?	iwakālua	5x4	lima kāuna (*)
3		kolu			30	10x3	kana-kolu		
4		hā		kāuna (*)	40	10x4	kana-hā	40	ka‘au*, ‘iako*
5		lima			50	10x5	kana-lima		
6		ono			60	10x6	kana-ono		
7		hiku			70	10x7	kana-hiku		
8		walu			80	10x8	kana-walu	2x(10x4)	lua kana-hā
9		iwa			90	10x9	kana-iwa		
10		‘umi			10^2		*hanele (haneri)*		
12	10+2	‘umi kūmā-lua	3x4	kolu kāuna (*)				$4x10^2$	lau
					10^3		*kaukani (tausani)*		
								$4x10^3$	mano
								$4x10^4$	kini
								$4x10^5$	lehu
					10^6		*miliona*		
								$4x10^6$	nalowale

Sources: Most terms are reported both in Elbert and Pukui (1979) and Pukui and Elbert (1986) as well as in Hughes (1982), except for the traditional term for 20, *lima kāuna* (reported in Hughes only).
Notes: Prefixes have been omitted for easier comparison. Loan words (in this case from English) are given in italics.
* Refers to a specific numeral (such as *ka‘au* in counting fish).

However, even a look at current dictionaries reveals distinct Polynesian lexemes reaching much further than 20—namely for the powers of ten times four (Pukui and Elbert 1986)—thus indicating both a mixed base system and a much greater extent of the indigenous system. As can be seen in Table 5, this system contained numerals for numbers as high as 4,000,000. Despite conceding the Hawaiians' "addiction to high numbers", Elbert and Pukui in particular doubt that they used these high numerals in a numerical sense, as "it is inconceivable that people counted that many" (1979:161). Instead, they consider the precise values attached to them as rather "fanciful" translations for words that actually were used to poetically indicate great numbers.

Besides *lau* (400) and *mano* (4000), the numerals under discussion are *kini* (40,000), *lehu* (400,000) and *nalowale* (4,000,000). While the first two are widespread among Polynesian number systems, the latter three are restricted to a much smaller area, although most of them still also appear outside Hawai'i. The term *kini* reflects *tini*, which can be found in Tahitian *manotini* for 10,000 (see Table 3), in Marquesan for 20,000 (Dordillon 1904, 1931) and in Māori where it is the numeral beyond *mano*, thus referring to an amount starting at 10,000 or to the number beyond counting (Best 1906:167f.). For *lehu* we find a cognate term in Tahitian, *rehu*, denoting 100,000 (Tregear 1969:207).

Whether or not the numerals so far refer to numerical values, the limiting number is achieved and labelled with *nalowale* at, allegedly, 4,000,000. *Nalowale* is translated by Elbert and Pukui (1979:161) as 'lost' and as merely signifying 'that the counter can go no farther'. In its meaning of 'out of sight' it has also been taken to convey the modern mathematical concept of infinity (Hughes 1982:254).

While the limiting number of the traditional Hawaiian system has produced some controversy, there appears to be widespread agreement about its original base. The literature we consulted states without exception that a mixed base 4 and 10 system was used before the missionaries' arrival (Alexander 1864:13, Elbert and Pukui 1979:161, Hughes 1982:255). Three arguments can be identified for such an exclusive use of the non-decimal system in pre-colonial times. The first dismisses the decimally composed lexemes for the tens as introduced by the missionaries; the second refers to specific lexemes for 4 and 40; and the third draws on the numerals for the higher powers from 400 onwards. However, a closer look at the numerals of both systems—in particular at 4, 40 and 80—casts some doubt upon such a general statement.

With regard to the first argument, we concede that 4 is emphasised from its earliest appearance in the number system. In addition to the common Polynesian numeral *hā*, a second lexeme (*kāuna*) can be found, which was used for counting tubers like sweet potato (Pukui and Elbert 1986:138). In at least some cases, higher number words were composed with *kāuna*—namely 3 x 4 (*kolu kāuna*) or 5 x 4 (*lima kāuna*)—instead of using the regular decimal numerals 12 (*'umi kūmā-lua*) or 20 (*iwakālua*) respectively.

The next power with a distinct numeral in a mixed base system should be 40 (= 10 x 4). While Elbert and Pukui (1979:159, following Alexander 1864:13) assume that the numbers for the tens above 50 were introduced, Hughes (1982:254) claims the same for the terms from 30 onwards. But Hughes's claim can be refuted if we consider the traditional term for 80, *lua kana-hā* (Elbert and Pukui 1979:162). If a mixed base 4 and 10 system is assumed to have been the traditional way of counting, a composite for 80 (as 2 x 40) should also consist of traditional elements; this is the case with *lua*, and most likely then also with *kana-hā*. But if we consider *kana-kolu* (10 x 3) and *kana-hā* (10 x 4) to be Polynesian, why should we regard the terms *kana-lima* (10 x 5) or *kana-hiku* (10 x 7) as not being Polynesian? Would it really be more likely for the latter to be composed as *kana-hā me kana-kolu* (10 x 4 + 10 x 3)? Even if we leave unanswered the question of the higher tens, composing 30 as *kana-kolu* (10 x 3) and 40 as *kana-hā* (10 x 4) clearly reveals a decimal principle.

The second argument for a base 4 and 10 system can be found in the distinct lexemes for 4 (*kāuna*) and 40 (*ka'au* and *'iako*). In order to support the general applicability of this mixed base system, the lexeme for 80 would be required to be composed of one of these traditional distinct numerals for 40, but it is not. Instead, *ka'au* is reported to have been used solely for counting fish, while *'iako* referred to barkcloth and canoes only (Alexander 1864:14). In addition, we may assume that the specific numeral for 4, and most likely its multiples, was restricted to counting tubers. But if these distinct lexemes of the indigenous system were indeed used for certain objects only, they indicate the existence of entire counting systems that are restricted to these objects and that supplemented a generally used system, as we have already seen in the case of Māori numeration.

For the higher numerals from 400 onwards, as cited in the third argument, we do not know whether their use was restricted to the respective objects, but it seems likely that these terms were used in continuation of the systems emphasising 4 and thus applying to the same categories. However, if we assume a parallel use of a decimal and at least one supplementary non-decimal system, why then do we find high numerals only in the non-decimal one? One reason might be that not all numerals were documented during the time of culture contact. Another more compelling argument could be that it is precisely the supplementary system that was concerned with achieving high numbers (cf. Bender and Beller n.d.a), as we will discuss in the conclusion.

There is some speculation regarding the reasons for using 4 as a secondary base. Kawena Johnson (cited in Hughes 1982:254) assumes that it originates from the main patterns of basket-weaving and in astronomy. Beyond this pragmatic reason, 4 was also of extreme significance in a spiritual context. Elbert and Pukui state in their *Hawaiian Grammar* that both 4 and 8 were formulistic numbers and that 8 was even "sacred when used as a suffix" (1979:161f.; see also Biggs 1990b). Yet, this finding again suggests that the counting systems applying 4 as supplementary base were restricted to certain, particularly significant objects. A third explanation is provided by Alexander (1864:13) who suggested that 4 as a secondary base goes back to the custom of counting those objects (i.e., fish, coconuts, taro and so on) by taking two in each hand or by tying them in bundles of four.

This custom again supports our conjecture that 4 was used as a counting unit within a decimal system rather than as a mathematical base. And we do indeed find indications that a decimal base was not at all unfamiliar and indeed in wide use before the missionaries' arrival. One of these indicators is syntactical in nature: only the numerals below ten are preceded by a general classifier *'e-* (Elbert and Pukui 1979:155). In addition, Hawaiians are described by Hughes as having been "used to thinking in terms of 'tens'. Their year was based on ten-day periods, *kana 'ēkā* was ten bunches of bananas, and *['o ka wa'a] kana ko'olua mai* was ten two-man canoes" (Hughes 1982:255, supplemented and corrected according to Elbert and Pukui 1979:159).

These findings rather suggest the parallel use of two systems instead of the exclusive use of one non-decimal system. The system with mixed bases—that is, a decimal system operating on 4 as a counting unit—might have been restricted to specific objects. Turning now to Rennell and Bellona, we find that a similar pattern can be identified and illuminated there to an even greater extent.

Number Systems on Rennell and Bellona

Rennell (Mugaba) and Bellona (Mungiki), situated in the southwest of the Solomons, are Polynesian Outliers. The languages spoken in these two neighbouring islands differ only slightly and are typically described together.

In general, the Rennellese counting systems (Table 6) reveal several characteristic features that they share with other Polynesian counting systems: They are basically decimal, they encompass a general system and various systems for specific objects and they have high numerals, the highest one being *nimo* = 1,000,000 (Elbert 1975, 1988:189).

Most numerals of the general counting system resemble—within the ranges of regular sound shifts—common Polynesian stock: the numerals for 1 to 9, *angahugu* for 10 and *gau* for 100. However, some of the higher Polynesian numerals (such as *ahe*, *mano* or *tini*) reappear in the specific systems. An alternative term for 10 (*katoa*) and the terms for higher numbers—*noa* (1000), *bane* (10,000), *tuia* (100,000) and *nimo* (1,000,000)—diverge from other Polynesian systems (see Table 3).

To summarise, the specific counting systems distinguished by Elbert basically relate to the following categories (for more details, see Elbert 1988:192-95):

C1	animates like humans, gods, large fish, mammals or birds
C2	smaller fish
C3	crustaceans, octopuses and eels; layers, strands and walls
C4	long objects like trees, rattan pieces, arrows, fish hooks, paddles or boards
C5	spears
C6	flat objects such as mats, rolls, bags, leaves, piles of cord, gravel or paper
C7	thatch panels
C8	canoes
C9	coconuts and other round objects
C10	bananas
C11	yam and breadfruit
C12	*panna* yam, topped taro (for pudding) or sweet potatoes
C13	taro stalks (untopped, including stems, leaves and tubers)
C14	fathoms, *sogo* and *masi* puddings and bags of taro tubers

In addition, Christiansen (1975:18) identifies a specific counting system in Bellonese for garden divisions.

Categorising these systems on the basis of similar principles, we can divide them into two groups. The first group, including categories 1 to 8 (with C5 uncertain, as insufficient data is available), is rather close to the general system except for the usage of numeral classifiers. Classifiers distinctive for all these categories precede the numeral *angahugu* (10) when referring to ten and replace it in all multiples of ten. In some categories, the same (in rare cases a different) classifier also precedes the digits below ten, and in some categories it even precedes the numeral *gau* (100) in terms for hundred and its multiples. Only in one category (C7) is the term *gau* replaced with a new classifier. From 1000 onwards, counting follows the general pattern, with the numerals *noa* (1000), *bane* (10,000) and *tuia* (100,000) confirmed.

Table 6: General counting and irregular categories in Rennellese/Bellonese.

	General Numerals	C9 Coconuts	C10 Bananas	C11 Yam, Breadfruit	C12 Root crops	C13 Taro stalks	C14 Taro tubers*
			[piles, 4 bunches each]	[pairs]	[piles, baskets]	[bunches, 5 in each]	[bags]
1	tasi/tahi	tau D	D nga ga‘akau	D	D	toka-D	goha D
2	gua	tau D	D nga ga‘akau	D	D	toka-D	goha D
3	togu	tau D	D nga ga‘akau	D	D	toka-D	goha D
4	hā	tau D	D nga ga‘akau	D	D	toka-D	goha D
5	gima	tau D	D nga ga‘akau	D	D	toka-D	goha D
6	ono	tau D	D nga ga‘akau	D	D	toka-D	goha D
7	hitu	tau D	D nga ga‘akau	D	D	toka-D	goha D
8	bagu	tau D	D nga ga‘akau	D	D	toka-D	goha D
9	iba	tau D	D nga ga‘akau	D	D	toka-D	goha D

continued over

	General Numerals	C9 Coconuts	C10 Bananas	C11 Yam, Breadfruit	C12 Root crops	C13 Taro stalks	C14 Taro tubers*
			[piles, 4 bunches each]	[pairs]	[piles, baskets]	[bunches, 5 in each]	[bags]
10	katoa, angahugu	**hīniu**	**‘āsea**	**kau**	**tini**	matā angahugu	**kumi**
10^2	gau	**tehua**, gau	**mano**	**kauhusi**	**ahe**, gau	gau	gau
10^3	noa	**mano**	noa	**ahe**	noa	noa	**kiu**
10^4	bane	**kiu**	bane	noa	bane	**nonoa**	bane
10^5	tuia	tuia	(?)	bane	(?)	(?)	(?)
10^6	nimo						
L	10,000,000	1,000,000	100,000	1,000,000	100,000	100,000	100,000
A	10,000,000	1,000,000	400,000 bunches (~ 4,000,000 bananas)	2,000,000	1,000,000 (yam) 400,000 (taro)	500,000	?

Source: Adapted from Elbert (1988:186-200). Following our principal source (Elbert 1988:190f.), we assume that the Rennellese and Bellonese systems largely coincide. However, the Bellonese counting systems provided by Christiansen (1975:18f.) do partially differ. The most notable divergence—if it is not a typographical error—is a gap between 1000 and 100,000 in Bellonese. This gap involves a numerical value for *bane* (in the general system and in C2) or *kiu* (in C9) of 100,000 instead of 10,000. Correspondingly, the highest numerals of Elbert's list—*tuia* (100,000) and *nimo* (1,000,000)—are missing in Christiansen's list. In addition, *ahe* is used instead of *gau* for 100, not only in C12 but also in the general system and in C11. *Notes:* For easier comparison, "D" stands for the digits 1-9 in all columns except for the general one, and higher numerals not reflecting the general pattern are emphasised. "(?)" indicates that Elbert's informants were uncertain. "L" gives the limiting number of each system, and "A" the absolute amount of single items as calculated from L and the counting unit.

*Also used for fathoms and *sogo* and *masi* puddings.

The counting systems that are of greater interest for our purpose are those of the second group, encompassing categories 9 to 14 (cf. Table 6), which are marked by irregularities and bear some similarities with specific counting systems in other Polynesian languages.

While in the general counting system, the absence of higher Polynesian numerals (see Table 3) is striking, most of the irregular counting systems apply at least one of these Polynesian terms: most notably *mano* in C9 and C10, *ahe* (= *afe* in Samoan and Tongan) in C11 and C12, *tini* (= *tini* in Marquesan, Tahitian and Māori, or *kini* in Hawaiian) in C12 and *kiu* (presumably *kilu* in Tongan or *'iu* in Tahitian; see Clark 1999) in C9 and C14. Some counting systems even apply numerals that are used only in specific counting systems in Tongan (see Bender and Beller n.d.a): *tehua* in C9 corresponding to *tefua* ('10 scores of coconuts'), *kau* in C11 corresponding to *kau*/*tekau* ('20 coconuts, fish or yam') and *kumi* in C14 corresponding to *tekumi* ('10 fathoms').

More remarkable than these Polynesian reflexes are the changes of numerical values that appear in these systems. Some of the systems (i.e., C10 to C14) do not refer to single items, but rather to sets of items, such as pairs, bunches, piles, bags or baskets of crops. Yet, they do not follow a common pattern (Christiansen 1975:17, Elbert 1988), but are counted as follows:

- bananas in piles containing four bunches each (C10),
- yam and breadfruit in pairs (C11), with ten pairs in a basket,
- *panna* yam in piles, with ten in each, or eight if they are large (C12),
- topped taro for pudding in baskets (C12), before conversion usually four pieces in each,
- untopped taro stalks in bunches of five each (C13) (according to Christiansen (1975:17), bunches of *ngeka* taro contain 12, of *sua* taro 22),
- taro tubers in bags (C14).

Even coconuts, the only category in this group typically counted as singles, when husked may be put on strings with ten nuts per string (Elbert 1988:193). One object of the first group also applies a diverging counting unit: rolls of pandanus leaves to be used for thatch, being counted as flat objects in C6, usually contained 60 or 72 dried leaves (Elbert 1988:194).

In order to elucidate the implications of operating with different counting units, we translate the example of 7600 piles of bananas from Elbert's (1988) introduction:[4]

hitu-nga	noa	(toe)	ono-nga	mano
7-fold	1000	(+)	6-fold	hundred [piles of bananas]
			= 7,600	piles of bananas [4 bunches each]
			= 30,400	bunches of bananas

Further assuming ten single bananas in each bunch, this offering might have roughly totalled 300,000 bananas.

The limiting number, which is clearly marked in the categories of the regular group with the next power beyond *nimo* (resulting in L = 10,000,000), is somewhat blurred in the irregular group. For all categories except C11, informants are reported to be uncertain with regard to the numeral for 100,000, and in C13 the same numeral is even given for 10,000 as for 100,000 (*noa*, *nonoa*). In C11, *noa* and *bane* (typically referring to 1000 and 10,000), are used for 10,000 and 100,000 respectively, thus indicating an extension of the number system beyond its usual limiting number.

If we consider the absolute number of single items referred to, the largest amounts range between 400,000 and at least 4,000,000 (see last row of Table 6). Again, the question arises as to how seriously these high numerals should be taken from a numerical perspective. This time however, Elbert himself—despite still claiming that the large numbers used in food distribution "have never been taken too literally but symbolise unfathomably large quantities" (1988:187)—provides evidence to the contrary. He notes the "importance attached in the old culture to planting, fishing, and ostentatious display of religious zeal" (1988:192) and an "emphasis on carefully counted quantity" (1988:186,198). The context of counting is described in a way that leaves no room for doubt; counting was indeed important and particularly so *before* Western influence (Christiansen 1975:63, Elbert 1988).

While we can assume that in general divergent counting systems, probably connected to a range of numeral classifiers, is a pan-Polynesian trait (Bender and Beller n.d.b), the elaboration of counting methods on Rennell and Bellona arose from the "cult of public generosity" (Elbert 1988:198). Fishing and gardening were not only the basis of subsistence, but also before conversion in 1938 the basis of a chief's prestige. The more he had to offer to the gods and his people, the higher his status. On Rennell and Bellona, Polynesian and Melanesian traits—that is a chief's role of collecting and redistributing goods and the public display of generosity more typical of Melanesian big men—were fused. This aspiration to reputation culminated in the *sanga hetau* 'planting competitions' (see Christiansen 1975:63). A *sanga hetau* required huge amounts of food as well as careful counting, which was always observed by crowds of people. One of the last big competitions yielded 10,000 coconuts and 7600 piles of bananas (Elbert 1988:186), which—with four bunches in each pile—totalled 30,400 bunches of bananas.

Although feasts were still celebrated after conversion to Christianity, planting competitions and offerings were banned by the missions (Christiansen 1975:63, Elbert 1988:187f.). Accordingly, the traditional counting system is largely forgotten now (Elbert 1988:186), and even when Elbert collected his data in the late 1950s and early 1960s, informants did not remember all the details.

Specific Counting Systems—a General Polynesian Pattern?

Although Māori, Hawaiian and Rennellese contain some of the most interesting cases of specific counting systems, they are not the only Polynesian languages that show evidence of such systems. For at least two other languages, Tongan and Samoan, a similar use of specific counting systems is documented. Both languages have regular base ten systems for counting "ordinary" things. In addition to these general systems, we find peculiarities with regard to certain objects: several classifying particles to be used with numerals when counting these objects and even specific modes of counting in certain cases.

Samoan has 15 different numeral classifiers that are required when counting food (Mosel and Hovdhaugen 1992). While most of the classifiers merely specify the adjoined numeral, several also change its numerical value (see Table 7).

A corresponding change in numerical value could also be found in the four specific counting systems in Tongan. Supporting a general decimal system, the specific systems were, again, restricted to certain objects (see Table 8). Despite bearing some resemblance to old classifiers, the number terms used in the specific systems generally functioned as numerals, which defined apparently "mixing bases". Counting started with pairs and continued either in tens of pairs, or in scores and tens of scores (see Bender and Beller n.d.a for more details).

Table 7: Numeral classifiers with multiplying effect in Samoan.

Object	Classifier	Operation	Example with 2 *(lua)*		
coconuts, young pigs	-*oa*	**x2**	*luaoa*	= 2-*oa*	**= 4**
skipjack	-*ʻaui*	**x10**	*luaʻaui*	= 2-*ʻaui*	**= 20**
coconuts	-*aea*	**x20**	*luāea*	= 2-*aea*	**= 40**

Sources: Adapted from Milner (1966), Mosel and Hovdhaugen (1992:246-50).

Table 8: Specific number systems in Tongan.

Object		Counting unit	Example with 2 *(ua)*		
sugar cane	**2**	*nga'ahoa*	*ua nga'ahoa*	= 2 pairs	**= 4**
coconuts, pieces of yam and fish	**2**	*taua'i* / *nga'ahoa*	*taua'i… 'e ua* / *ua nga'ahoa*	= 2 pairs	**= 4**
	20	*tekau* / *kau*	*uangakau* / *kau… 'e ua*	= 2 scores	**= 40**

Sources: Adapted from Bender and Beller (n.d.a).
Notes: Although similar with regard to their counting unit, the objects of the second category (i.e., coconuts, pieces of yam and fish) are counted with diverging systems, which differ in syntax and in the terms for 'pair', 'one score' and 'ten scores'.

It is plausible—although there is insufficient data to prove it—that similar specific counting systems also existed in parts of the Cook Islands (Large 1902) and in the Marquesas (Dordillon 1931, Lemaître 1985), where the higher numerals referred to powers of 10 times 2 (in the southwestern group) or times 4 (in the north-eastern group). More conclusive data is also available for Tahiti and Mangareva (Lemaître 1985). In Tahiti two systems were in use, one regularly decimal, the other applied to pairs of coconuts, breadfruits, bonitos, pandanus and thatch, and in Mangareva four different systems seem to have applied either 1, 2, 4 or 8 as the counting unit, depending on the object counted. According to Clark (1999:197f.), extensions of the general number system had indeed occurred in all Polynesian languages, varying in scope with population size, wealth and social stratification.

OBJECTS AND OBJECTIVES OF SPECIFIC COUNTING

The Polynesian languages examined so far share some interesting characteristics. In spite of regular claims to the contrary, they all contain general number systems that are clearly decimal in nature. All three languages we examined in detail also contain additional number systems, which are restricted to certain objects and seem to mix a different fundamental base together with 10. Is it possible to identify common patterns among these mixed base systems or the specifically counted objects across languages? And why do these systems often go together with numerals for high numbers?

Mixed Bases or Counting Units?

What the five languages considered have in common is different ways of counting different items. However, when examining the characteristics of the way in which they do this there appear at first glance to be more differences than similarities. Some of the languages apply numeral classifiers that multiply the adjoined numeral (e.g., Rennellese and Samoan), some implicitly refer to different counting units depending on the counted objects (e.g., Māori and Rennellese) and some use different numerals altogether for different objects (e.g., Hawaiian, Rennellese and Tongan). In addition, the multiplications taking place do not follow a consistent or coherent pattern. In view of this, can we still hope to find a common principle behind these peculiarities?

Clark identified a tendency of those classifiers that precede the numeral to have a multiplying effect, often described as "counting by groups of ten" (1999:198). However, he also emphasised that the effect is "a little more complicated than this" (1999:199). Christiansen noted (1975:17) a "custom of differential counting of aggregated objects" in Rennellese, which he assumes might be derived from "an old system of equivalent 'values'". If he had an old pan-Polynesian system in mind, the findings reported here seem to contradict his assumption. Of the equivalencies identified by Christiansen—notably counting 2 *'uhi* yam, 10 (sometimes 8) *'uhingaba* yam and 12 (sometimes 22) taro as one each—only one can be found elsewhere in the Polynesian triangle, namely the pair of yam in Tongan. Other than this, factors differed widely and rather unsystematically (see Table 9).

Scholars typically only recognised the concern with 20 in many Polynesian languages, often denoted with the same specific term *tekau*, and this led them to speculate about vigesimal systems (Best 1906:158, Large 1902, Smith 1902:216, Tregear 1969:503f.). However, a cognate of the term,

Table 9: Mixed bases and diverging counting units in five Polynesian languages.

	Counting units							
Language	2	4	5	8	10	12	20	22
Rennellese	x	x	x	(x)	x	(x)		(x)
Samoan	x				x		x	
Tongan	x						x	
Māori	x							
Hawaiian		x						

ka'au, denotes 40 in Hawaiian (Alexander 1864:14), and in other Polynesian languages 20 is not the only and not even the most important factor, as Table 9 shows (and see Lemaître 1985).

Nevertheless, we do agree with Christiansen (1975:17) that the aggregated counting of larger sets reflects a general Polynesian concern. Hence, we suggest that these systems should be regarded not as mixed base (in the sense of "semi-vigesimal"), but as decimal systems that operate on different counting units. This interpretation is supported, for instance, by the lexeme *tekau* itself. It is derived from *te kau*, which means 'the collection, assemblage' (Best 1906:159,162; and see Tregear 1969:503), 'the group' (Elbert 1975) or 'the tally' (Williams 1988:411). Instead of being a genuine numeral, it may therefore be regarded as a "countable base" (see Harrison & Jackson 1984) or, more generally, a unit for counting (Lemaître 1985). Additional support is provided by the range of diverging counting units within one language (as in Rennellese or in Tongan). If one of these languages really did apply a mixed base system, we should expect the same mixing base in all its systems. Finally, the order of the number words themselves in all of these specific systems remains perfectly decimal; only the absolute amount of items they referred to was multiplied by a specific factor (cf. Bender and Beller n.d.a, Christiansen 1975:17).

Specifically Counted Objects

Specific number systems were often bound up with traditional practices of food production, feasting and religious ritual (Clark 1999:198). Given their vast distribution throughout Polynesia, it seems plausible to assume that certain objects have always been dealt with specifically, either by classifiers in counting or by different counting systems. This conjecture is supported by comparing the objects that were counted specifically.

Rennellese, containing 14 different classifiers altogether, applied diverging counting units at least to bananas, yam, breadfruit and taro (and in a restricted sense also to coconuts and pandanus leaves). These objects largely coincide with the group of food plants for which honorific terms exist; besides *Santiria apiculata* and tree fern, these are coconut palms, bananas, yam and taro (Elbert 1988:146f.). Given that Rennell and Bellona had a pronounced taro culture (Elbert 1988:193), the emphasis on taro, for which three different counting systems had even been in use, is not surprising. In Samoan, the category of objects that required classifiers comprised different sizes of fish and other seafood, birds, pigs, coconuts, taro and yam, breadfruit and bananas (Milner 1966, Mosel and Hovdhaugen 1992); the classifiers implying multiplication refer to fish, young pigs and coconuts.

The other three languages use numeral classifiers only for humans (and sometimes for other animates), but made more systematic use of the multiplication effect. The specific counting systems in Tongan based on pairs and scores were applied for fish, coconut, yam, sugar cane thatch and pandanus leaves for weaving. Fish, birds and tubers were also among the objects that are documented instances of the exceptional dual mode in Māori. And in Hawaiian, specific numerals involving four are reported for fish, barkcloth, canoes and tubers. To summarise this enumeration: in four of five languages, fish and the most prestigious tubers belonged to the category of specifically counted objects, and in most of them coconuts and material for fabrics were also included (see Table 10).

In each language, the respective objects were traditionally of particular cultural importance. No single object was shared by all languages, but they all followed the same principle. The structural similarities between the specific counting systems, the linguistic relationship between numeral classifiers and specific numerals, and the general range of specifically counted objects all indicate a common pattern, if not a common source. We argue that these systems reflect the same concern in most Polynesian languages and that the variance in detail is not only plausible, but a conclusive consequence of this concern. What all these objects have in common is the fact that they are subsistence products that were both abundant and culturally important (see Bender and Beller n.d.a). It is precisely this combination of features that characterises the supplementary use of number systems with diverging counting units. In most cases these systems go together with numerals for high numbers and, as we will show now, for good reasons.

Expanding the Limiting Number

With regard to the highest numerals, there is some linguistic evidence that they were also used numerically. One piece of evidence is that many of these terms have equivalents in other Polynesian languages, with varying, but usually high values. The Proto-Polynesian terms **rau* (100), **afe* (1000) and **mano* (10,000)—and, according to Clark (1999:197), probably even **tini* (100,000) and **kilu* (1,000,000)—support the thesis that the Polynesian system initially extended up to a limiting number of at least 100,000 (if not 10,000,000).

The variety in contemporary languages may result from expanding or contracting this system according to local requirements. When considering the type of objects that were counted specifically in at least some of these languages, an emphasis on resources that were both culturally significant and abundant becomes apparent. One of the local peculiarities that might

Table 10: Objects of particular concern—as indicated by numeral classifiers (NC) or specific counting systems (CS)—in five Polynesian languages.

	Rennellese		Samoan		Tongan		Māori		Hawaiian	
Category	NC	CS	NC	CS	NC	CS	NC	CS	NC	CS
human	x		(x)		x		x	x	(x)	
fish (or certain species of fish)	x			x	x	x		(x)		x
shellfish, crabs, lobster	x		x		x					
birds / fowl	x		x		x			x		
pigs	x			x	x					
coconuts	x	(x)	x	x		x				
yam or taro or sweet potato	x	x	x			x		(x)		(x)
breadfruit, bananas	x	x	x			(x)				(x)
pandanus leaves or tapa	x	(x)				x				x
thatch (of sugar cane)	x					x				
canoes	x									x

Sources: Instances from Tongan are taken from Bender and Beller (n.d. a) and Churchward (1953), for Samoan from Mosel and Hovdhaugen (1992:246-50), for Rennellese from Elbert (1988), for Māori from Best (1906) and for Hawaiian from Alexander (1864) and Hughes (1982).
Notes: Specific ways of counting that change numerical values are underlaid in gray. Additional classifiers not included here refer to long objects, flat objects, fathoms/puddings and spears in Rennellese, and to wrapped packages of food in Samoan.

have given rise to expanding or contracting the extent of the system could therefore have been the size of the population and the degree of stratification. In islands with powerful chiefs or kings (such as Tonga, Tahiti or Hawai‘i, and probably Rennell), concern with collecting and redistributing resources was strong (see Kirch 1984, 1986; Martin 1991:115). Accordingly, the quantities of resources, the provision required for war parties or the material needed for traditional fabrics, to mention just some of the most salient, inevitably yielded high numbers and necessitated high numerals. Societies with less centralised political forces or small communities (such as Māori or some of the Outliers), on the other hand, might not have needed the very large numbers.

In our introduction we cited Elbert and Pukui’s doubt that the high numerals were used for high numbers since they consider it inconceivable that people counted that far (1979:160f.; see also Clark 1999, Elbert 1988:187). However, it is not the counting process that yields high numbers and thus requires high numerals, but rather calculation. And it was calculation that was most probably applied during the collection and redistribution of resources. In order to keep track of the movements of barkcloth, mats, fish and other goods, people with skills in computation were in high demand, as Hughes (1982:254) argues for Hawai‘i. The same was necessary when providing big war parties with food (see Martin 1991:115). In even more conclusive detail, Elbert (1988) and Christiansen (1975) describe the competitive giving (*sanga hetau*) on Rennell and Bellona that not only required huge amounts of food, but also necessitated their careful calculation and computing.

For these experts, who did not have a notation system, dealing with large numbers presented difficulties. And it is exactly in this context of accounting where specific counting systems make sense. In extracting a certain factor (such as 2, 4, or even 20) from the absolute amount, these numbers could be abbreviated and the cognitive effort required to operate with them facilitated (Beller and Bender 2005). It is therefore no coincidence that particularly in languages with high numerals—and we may add, with a concern for high numbers—supplementary systems were in use as well.

Numbers appeared not only in resource collection and redistribution, but also in genealogies. In these contexts, some Polynesian cultures also displayed a concern with high numbers. In Hawai‘i this is apparent in the Kumulipo genealogical chant (Elbert and Pukui 1979:161). In Tonga, the Tu‘i Tonga line can be traced back for 1000 years, thus encompassing close to 40 generations (Campbell 2001:264-66). Also for Māori, this context was significant because counting generations in genealogies was the only instance for which an external representation, consisting of a piece of hardwood marked with notches on one side, is reported (Best 1906:170, 1921:71). If we add historical accounts referring to, for example, the size of war parties (Bauer *et al*. 1997:284, Elbert and Pukui 1979:160f.), we might concede that in these social contexts high numerals were indeed used in a rather poetic way or even to intentionally exaggerate the absolute amount, as this might have added status and prestige to the chief concerned. Applying strict numerical values to these terms might then strike us as "fanciful" translations.

However, different from this socially motivated use, it can be assumed that the accounting of resources was concerned with accurate numbers. And here, we believe, the numerals did indeed retain genuine mathematical values of great extent.

CONCLUSION

An internal comparison of Polynesian number systems as well as comparison with number systems of other Austronesian languages, both present and reconstructed, not only highlights the considerable degree of cognacy among the numerals. Comparison also reveals some common patterns: these systems were basically decimal and most of them extended well beyond 1000. The commonalities thus found support the conclusion that the Oceanic-speaking ancestors of the Polynesians brought with them a number system with base 10 and extending to (at least) 1000. Despite the regular character of their general system, however, all five languages examined here also contained, in pre-European times, distinct counting systems with bases that were not strictly decimal. Some placed an emphasis on 20, but none was vigesimal. Instead, the systems can be characterised as decimal systems operating with diverging counting units, in most cases involving one or more of the factors 2, 4, 10 and 20. The crucial point, though, is that these number systems with diverging counting units were not used exclusively, we argue, but they supplemented more general systems with strictly decimal bases.

It is often argued that specific counting systems are predecessors of general and abstract number systems (Ifrah 1985, Menninger 1969). Even if this assumption is true on a large scale, it does not justify regarding all systems restricted to specific objects as cognitively deficient. Some might have been established as a reasonable solution to a practical problem, in this case, the problem of how to deal with great numbers when no notation is available. The specific systems supported this endeavour in two ways: in "transposing" the general system onto a higher level, they both extended the limits of the general system and facilitated dealing with large sets of objects (Beller and Bender 2005). Therefore, specific systems in general—and the systems with pairs or scores as counting units in particular—can in fact be cognitively advantageous.

The analysis presented here provides a plausible explanation for the existence of such specific counting systems in Polynesian languages, namely an intentional adaptation of the general decimal system. It should be noted, however, that it is also conceivable—and has been argued (Harrison and Jackson 1984)—that this development could have happened unintentionally through mere linguistic change. Some of the specific number terms of Polynesian languages, for instance Tongan *tefua* ('200 coconuts'), have cognates in the related Micronesian languages, where they reflect numeral classifiers that date back at least to Proto-Micronesian and probably even Proto-Oceanic: PMC *-*ua* for general objects, from POC **pua* 'fruit' (Jackson 1986:209, and see Bender and Beller n.d.b, for a more detailed discussion).

While we cannot prove that the specific counting systems were developed on purpose, instead of having evolved linguistically out of such classifier systems, we still consider it more likely. They were derived from the general system and they were adapted to environmental and cultural conditions. The objects to which they were restricted share abundance and significance. They implied a numerical operation, namely the multiplication with a counting unit other than one; and the counting unit predominantly applied—the pair—is easy to deal with, both practically as well as cognitively. In addition, this counting unit seems to reflect a larger Polynesian cultural pattern emphasising symmetry (Alexander 1864, Bender and Beller n.d.a, Best 1906, Elbert and Monberg 1965:vii, Hanson 2004, Lemaître 1985). At least in the Polynesian case, the genuine interest in high numbers, particularly in highly stratified societies, and the cognitive constraints set by the lack of notation provided the right context for such a cultural advance.

More than a century ago, Tregear (1892:56) stated that decimal aspects of Māori number systems must have been a late development "for we cannot allow that any rendering of decimal notation is possible to primitive savage peoples, whose difficulty in getting beyond any numerals above 3 and 4 is well known". Quite contrary to this long outdated stance, we propose that the Polynesians did indeed have elaborate, regular and extensive decimal systems. Furthermore, under the unfavourable conditions of mental arithmetic without notation, their calculation experts apparently also developed strategies for dealing with high numbers in a very cognitively efficient way.

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NOTES

1. This account of the Austronesian migration seems to be the most substantiated one. There are alternative models (e.g., Oppenheimer 2004, Terrell 1986). However, this controversy does not affect our main topic, with regard to the colonisation of Polynesia, where the competing models are highly congruent.
2. Among the South Island Māori in the 1840s, *hoko*- may have been used differently (Harlow 1987:19). The composition of *hoko*- and a single numeral *n* (e.g., *hoko-whā* [*hoko*-4]), is analysed as 20+10 x n (i.e., 20+10 x 4 = 60). If this analysis were correct, it would provide a fascinating exception to the rule; however, the data available is not sufficient to allow a thorough reanalysis. We thank one of our anonymous reviewers for this suggestion as well as for two other references, Kendall (1815) and Maunsell (1842), which unfortunately were not available for cross-checking.
3. Best quotes Large (1902) for a similar system on Aitutaki in the Cook Islands. However, a closer look at this report reveals that *oko*- was prefixed to the numeral, by which *takau* (20), even when omitted, was multiplied—as is evident from the "singular" form *okotai takau* = '*oko*-1 x 20' = 20. It should therefore rather be glossed as a causative 'multiplying by' or 'making x-fold'.
4. To a certain extent, this arithmetical example remains hypothetical, as from Elbert's (1988) own description of the same feast, it is not entirely clear whether the correct number was 700, 7000 or 7600, and whether it referred to bunches or piles. However, for the sake of illustration, we consider it justified to take the largest number mentioned.

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